NON-PARAMETRIC DETECTION IN UNDERWATER ENVIRONMENTS

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ABSTRACT

Motivated by the recurring use of the generalized Gaussian family to model different underwater noise sources and the asymptotic performance levels of some commonly used detectors for this family, we examined the performance of these detectors for several different underwater noise sources. The sources considered are non-Gaussian, highly correlated, generally non-stationary, and vary from being lighter to heavier tailed than Gaussian. Although the linear detector had the best performance, the linear rank detector and the L-level uniform quantizer consistently had similar performance levels. Considering the simplicity and the robustness of the quantizer and the rank detectors, either of these detectors would seem to be the best choice for these environments.

1. INTRODUCTION

Consider the simple binary hypothesis test:

$$H_0 : \underline{X} = \underline{N} \qquad H_1 : \underline{X} = \underline{N} + \underline{s} \qquad (1)$$

Let $\underline{X}$ represent the M-vector of samples received by the detector, $\underline{N}$ the noise samples, and $\underline{s}$ the samples of the known, positive signal. For the following analysis, let f_M denote the M-variate density of the noise samples $\underline{N}$. Define the performance parameters of a detector to be

$$\alpha = \text{Probability (decide } H_1 \mid H_0 \text{ true)} \qquad (2a)$$

$$\beta = \text{Probability (decide } H_1 \mid H_1 \text{ true)} \qquad (2b)$$

The parameter α is defined as the level or the false alarm probability, while the parameter β is the power or the performance level. A detector is said to be optimal if for any level $\alpha \leq \alpha_o$ the power β is maximized. As this is recognized as the Neyman-Pearson (NP) criterion, the resulting optimal (parametric) detector will be [1]

$$l(\underline{X}) = \log \frac{f_M(\underline{X} - \underline{s})}{f_M(\underline{X})} \quad \begin{matrix} \geq T_\alpha & \to & H_1 \\ < T_\alpha & \to & H_0 \end{matrix} \qquad (3)$$

where l is the log likelihood ratio, and T_α is the value of the threshold required to obtain a level α.

An alternative to (3) is to use a non-parametric detector. A detector is said to be non-parametric if its level α can be determined without knowledge of the specific distribution or functional form of the noise distribution. Strictly speaking, a detector which is non-parametric for white noise may fail to be non-parametric for correlated noise. All underwater environments to be considered are correlated, thus we will say a detector is non-parametric if the level α can be determined without knowledge of the functional form of the noise distribution, *provided the noise is white* [2-6].

It is clear that a non-parametric detection scheme is desirable if there exists little knowledge of the actual noise distribution. In addition, non-parametric detectors are generally more robust than parametric detectors and thus may be desirable for an environment which is non-stationary. The general structure of the detectors to be considered is a memoryless nonlinearity (ZNL) defined as l, followed by an accumulator and a threshold comparator.

2. BACKGROUND

2.1 The Data

Four different data sets were considered in this analysis. A summary of these environments is contained in Table 1, where the entries on the Shrimp, Merchant, and Seismic data sets have been taken from [7]. The largest data set under consideration is Arctic under-ice noise. The data, FRAM II, was recorded on April 23, 1980 at 11:30 - 11:40 pm. An omnidirectional hydrophone was suspended 91m in 4000m water and radio linked to an analog recorder. Subsequently, the data was analog filtered, 0.01 - 5 kHz passband, and lowpass filtered, 2.5kHz cutoff frequency. Finally, the data was digitized with a sampling frequency of 10 kHz [8]. Following Dwyer [8] and Veitch and Wilks [9], the data set was segmented into 6006 records with each record containing 1024 samples.

The second set of data was collected in the Pacific Ocean off the coast of Hawaii and is the sound of shrimp snapping their claws together. When the claws of the shrimp snap together high amplitude bursts which last from two to five milliseconds are created. The Merchant Shipping noise was recorded by a single microphone which was suspended in the Indian Ocean near a known merchant shipping lane. The final data, Seismic noise, was recorded on the continental shelf near a region of artificially-induced seismic activity.

Noise Source	Sampling Rate (kHz)	Length	Comments
Arctic	10.	6159144	non-stationary, non-Gaussian, four distinct modes
Shrimp	40.	95232	non-stationary, non-Gaussian, heavy tails
Merchant	1.25	102400	stationary, light tails
Seismic 1	0.892	95232	non-stationary, heavy tails
Seismic 2	0.892	124928	non-stationary, heavy tails

Table 1 - Summary of Underwater Noise Environments

2.2 Previous Work

Several different researchers have investigated the data sets under consideration. For the Arctic data, the analyses by Dwyer [8,11] and Veitch and Wilks [9] were used as a basis for this work. Dwyer concluded that the Arctic data was both non-stationary and non-Gaussian. In addition, he proposed a technique which could be applied to the Arctic data to separate wide-band and narrow-band processes from each other.

In 1985 Veitch and Wilks [9] analyzed the Arctic data. They agreed with Dwyer that the data was non-stationary. However, their analysis led them to conclude the data could be modeled as a mixture process which contained a Gaussian background component. They also examined the sample variance (with respect to the global variance), the sample skew, and the sample kurtosis of each record and concluded that the records could be grouped into four distinct categories.

Nielsen and Thomas [12,13] proposed an alternative univariate model for the data and examined memoryless parametric detection using the model. The correlation between samples was analyzed, and the performance of some detectors which incorporated the correlation of the data samples were analyzed and found to have a superior performance level to the memoryless detectors. In addition, an adaptive pre-processing algorithm was implemented which resulted in a further improvement.

A primary investigator of the Shrimp, Merchant, and Seismic data sets under consideration is Willett [7]. For all data types he proposed univariate models and examined parametric detection. Bouvet and Schwartz [14] used the same data sets as [7], but were concerned primarily with fitting the data to the Gauss-Gauss mixture and to Middelton's Class A models. In addition, Machell, Penrod, and Ellis [10] have recently claimed that the generalized Gaussian model is valid for Ambient, Shipping, Seismic, and Antarctic ocean noise.

2.3 Univariate Model

The generalized Gaussian family, which was first introduced by Kanefsky in 1965 [3], has become a popular model for underwater noise sources. Much of the interest has be due to the ability to model symmetric processes which are lighter or heavier tailed than Gaussian. In addition, the family includes the Gaussian distribution and the Laplacian distribution. As defined by (4), the family has a parameter for the mean, μ, the variance, σ^2, and the tail weight, c.

$$f(z) = A\ e^{-\|a\ |z-\mu|\|^c} \qquad (4)$$

with

$$a \equiv \frac{1}{\sigma}\left[\frac{\Gamma(3/c)}{\Gamma(1/c)}\right]^{1/2} \qquad A \equiv \frac{ac}{2\ \Gamma(1/c)}$$

For $c = 2$ the density is Gaussian and for $c = 1$ the density is Laplacian. In general, if the shape parameter $c < 2$ then the density has heavier tails than Gaussian, and for $c > 2$ the density has lighter tails than Gaussian. Since the relationship between the shape parameter and the kurtosis is invertible for $c \in [1.4, 5]$, knowledge of the kurtosis allows determination of the shape parameter c.

3. NON-PARAMETRIC DETECTION

3.1 Motivation

As a result of investigations on the generalized Gaussian family and its suitability to underwater environments, the performance levels of four different detectors for the noise environments described have been examined. Consider a measure of performance between detectors i and j to be the asymptotic relative efficiency (ARE) [2]

$$ARE(i,j) = \frac{\xi_i}{\xi_j} \qquad \xi_i = \lim_{M \to \infty} \frac{1}{M}\left[\frac{\frac{d}{ds}E_{H_1}(i)|_{s=0}}{\sqrt{V_{H_0}(i)}}\right]^2 \qquad (5)$$

where ξ_i is the efficacy of detector i provided it is finite. If $ARE(i,j) > 1$ then detector i is superior to detector j.

The linear detector is used as a basis for comparison and is both optimal and locally optimal for Gaussian noise. The memoryless transformation l and the efficacy can be defined as

$$l(z) = \sum_{i=1}^{M} z_i \qquad \xi_{lin} = \frac{1}{\sigma^2} \qquad (6)$$

where σ^2 is the variance of the noise process. A second detector, which is commonly considered, is the sign detector. In addition to being the NP optimal (non-parametric) detector for zero median noise and the NP locally optimal test for Laplace noise, the sign detector is robust. Let u be the unit step function and f be the univariate density of the noise, then the ZNL and the efficacy for the sign test can be defined as

$$l(z) = \sum_{i=1}^{M} u(z_i) \qquad \xi_{sign} = 4\ f^2(0) \qquad (7)$$

The linear rank (Wilcoxon-Mann-Whitney) detector is a non-parametric detector which incorporates information about the relative sizes of each sample by assigning a ranking r which lies between one and M and is dependent on the magnitude of a sample with respect to other samples. The linear rank detector is more robust than the linear detector and it uses more information about each sample in forming the test statistic than does the sign detector. The ZNL and the efficacy for this detector can be written as [5]

$$l(z) = \sum_{i=1}^{M} r(|z_i|)\ u(x_i) \qquad \xi_{rank} = 12\left[\int_{-\infty}^{\infty} f^2(x)dx\right]^2 \qquad (8)$$

In addition to the linear, sign, and rank detectors, uniform and non-uniform quantizer detectors were also examined for three of the four environments. Since the results of the non-uniform quantizer were very similar to those of the uniform quantizer, only the uniform quantizer will be considered henceforth. Define the quantizer as follows

$$l(z) = \sum_{i=1}^{M} q(x_i) \qquad q(x_i) = l_j \quad \text{if}\ \ t_{j-1} < x \le t_j \qquad (9)$$

where L is the number of levels, l_j, $j = 1, 2, ..., L$ are the vertical break points, and t_j, $j = 0, 1, ..., L$ are the horizontal break points with $t_0 = -\infty$ and $t_L = \infty$. For a uniform quantizer with a vertical spacing of $Q \approx \sigma/5$ between levels, we have that $l_j = -[(L-1)/2 - j + 1]Q$ and $t_j = -[L/2 - j]Q$. As with the linear rank detector, the quantizer is more robust than the linear detector and retains more information in the test statistic about the distribution of the samples than the sign detector. For a constant signal and a symmetric, zero mean noise source, the efficacy can be written as:

$$\xi_{quan} = \frac{\left\{\sum_{j=1}^{L} l_j\ (f(t_{j-1}) - f(t_j))\right\}^2}{\sum_{j=1}^{L} l_j^2 (F(t_j) - F(t_{j-1}))} \qquad (10)$$

Where f is the univariate density and F is the cumulative distribution function of the noise. A derivation of the efficacy can be found in Kassam [2]. It is clear that if the number of levels is optimized for a particular application the quantizer will always perform as well as the sign detector. Thus we will not examine the ARE with respect to the quantizer detector. In addition, note that the linear detector and the uniform quantizer detector are parametric detectors.

For the generalized Gaussian family of densities the efficacies are well defined. Thus the ARE between two detectors can be computed and found to be

$$ARE(sign,\ lin) = c^2\ \frac{\Gamma(3/c)}{\Gamma(1/c)} \qquad (11)$$

$$ARE(rank,\ sign) = 3 \cdot 2^{-2/c}$$

$$ARE(rank,\ lin) = ARE(sign,\ lin) \cdot ARE(rank,\ sign)$$

Figure 1 contains plots of the ARE vs shape parameter c for different pairs of detectors. Notice that the sign detector is better than either the linear or rank detectors for heavy tailed noise, i.e. $c < 1.4$. When the noise is lighter tailed than Laplacian but heavier tailed than Gaussian, the rank detector appears to be the best. Finally, for noise which is Gaussian or lighter tailed than Gaussian, the linear detector is the best while the rank detector will have a comparable performance.

3.2 Simulation Techniques

A vector of length $2M$ is taken from the data and we form $X^0 = (x_1, x_3, ..., x_{2M-1})$ the received vector under hypothesis H_0 and $X^1 = (x_2 + s, x_4 + s, ..., x_{2M} + s)$ the received vector under hypothesis H_1. Note that the two data vectors X^0 and X^1 each have sampling rates which are half of the original sampling rate of the data. Since each data set examined was over sampled by a factor of at least two, for no signal the two data vectors are identically distributed though statistically dependent on each other. To compute α and β, the functions $l(X^0)$ and $l(X^1)$ are computed, compared to a fixed threshold, and scaled by the number of iterations.
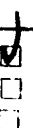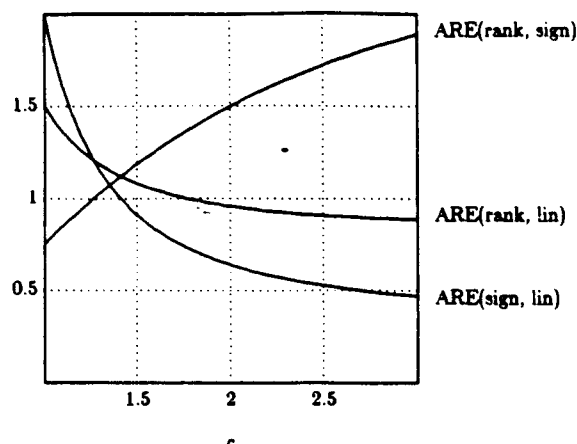
A-1

Figure 1 - ARE vs c

All of the data sets except the Merchant Shipping noise can be shown to be non-stationary. However, only the Arctic data can be shown to be composed of at least two different modes where each mode lasts a significant period of time. Thus, with the exception of the Arctic under-ice noise, the block size is defined to be the entire data set. In addition, the signal power is determined by the global variance of the data set, $s_i = \sigma/\sqrt{M}$. Simulations were made for a variable signal power and smaller groupings of records. However, all detectors had relative performance levels similar to those for the fixed signal power case with the only significant difference being a lower performance level overall. Since the Arctic under-ice data can be separated into four distinct modes, records belonging to the same mode were grouped together and examined separately. The signal power was defined on a record-by-record basis. If s_i is the ith element of the signal M-vector, then $s_i = \sigma_j/\sqrt{M}$, where σ_j^2 denotes the noise power of the jth record. For all simulations environments a signal length of sixteen was used.

3.3 Results

In Figure 2 the results of the simulations for the Shrimp noise are shown. Table 1 indicates that the noise is heavy tailed with respect to Gaussian, yet the sign detector has the worst performance. In particular, if the uniform quantizer detector is used, it is found that by using 16 levels the uniform quantizer performs better than the rank detector and nearly as well as the linear detector. In addition, notice that the performance level of the rank detector is similar to that of the linear detector.
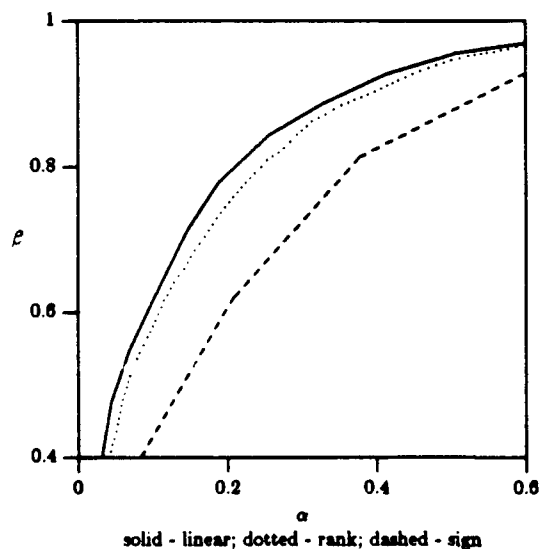

solid - linear; dotted - rank; dashed - sign

Figure 2. Performance Levels for Shrimp Noise

For the Merchant Shipping noise, the results were as expected. The shipping noise is lighter tailed than Gaussian, thus it is expected that the linear detector would have the best performance. Indeed, as shown in Figure 3, the linear detector is best though the performance of the rank detector is nearly identical to that of the linear detector. If the uniform quantizer detector is used, a small improvement over the sign detector can be achieved by use of 4 levels. However if 32 levels are used, the uniform quantizer and the rank detector have performance levels which are comparable.

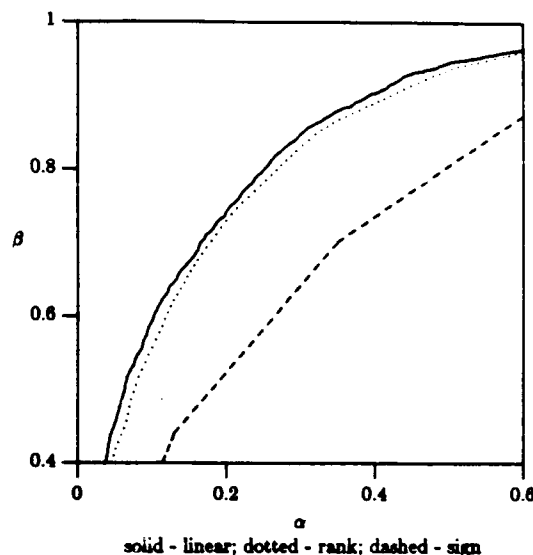solid - linear; dotted - rank; dashed - sign

Figure 3. Performance Levels for Merchant Noise

Both of the Seismic noise recordings have been shown to be heavy tailed with respect to Gaussian. Since the results for both seismic data sets were similar, only the results for the first data set will be presented. As shown in Figure 4, the sign detectors performance level, relative to the levels of the linear and rank detectors, has improved significantly. Indeed, for some levels α, the rank and sign detectors have performance levels better than the linear detectors. A uniform quantizer detector with 8 levels will perform as well as the rank detector for some ranges of α and will be comparable otherwise.
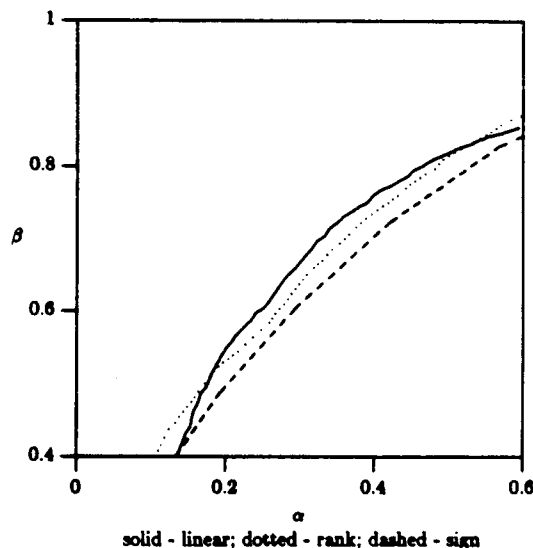

solid - linear; dotted - rank; dashed - sign

Figure 4. Performance Levels for Seismic 1 Noise

The Arctic data can be grouped into four different modes. The four modes can be defined either by the moments [8] or by the correlation function [9]. Both methods result in essentially the same groupings of records. Category 1, HSQL is characterized by records with a High variance (with respect to the global variance), a SQueezed or small skew, and a Low (less than three) kurtosis. Category 2, HSTN, is characterized by records with a

STretched or large skew, and a Normal (about three) kurtosis. Category 3, HSTL, is the same a Category 2 except these records have a Low kurtosis. Finally, Category 4, NNN, has Normal variance, skew, and kurtosis. For all modes, the linear detector had the best performance and the sign detector had the worst performance. Once again the rank detector performed nearly as well as the linear detector. Figures 5 and 6 shown the performance levels the the linear rank detector for the four different modes and the performance levels of the linear rank, linear, and sign detectors for the category NNN.
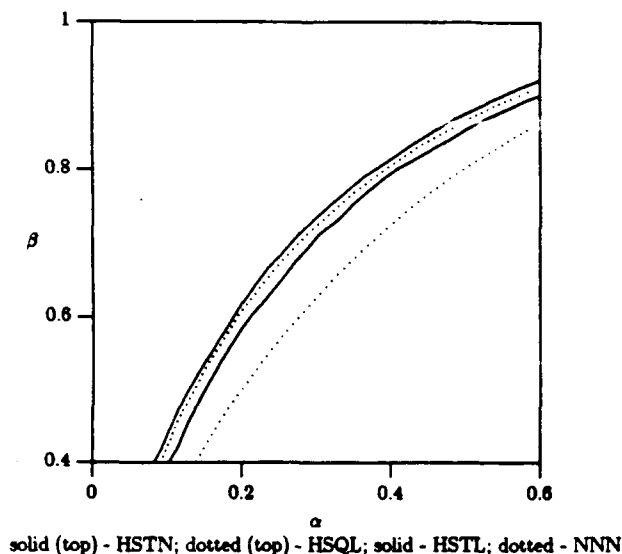

solid (top) - HSTN; dotted (top) - HSQL; solid - HSTL; dotted - NNN

Figure 5. Performance Levels for Linear Rank Detector for Arctic Noise
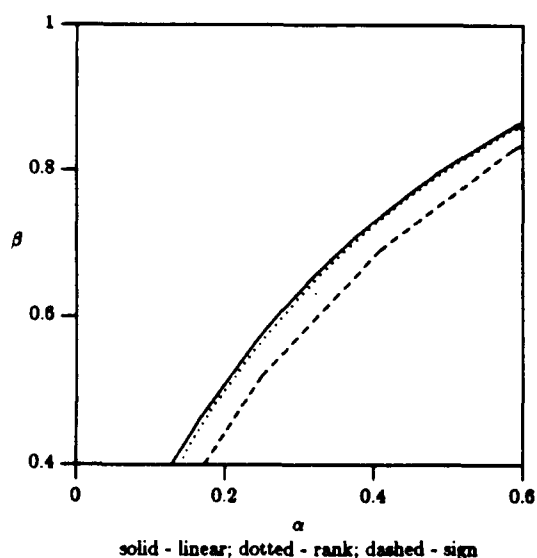

solid - linear; dotted - rank; dashed - sign

Figure 6. Performance Levels for Arctic NNN Noise

CONCLUSIONS

The results indicate that, while the linear detector almost always has the best performance level, the linear rank detector has a performance level which is similar, in general. In addition, a L-level uniform quantizer, which is simpler than the rank detector, can have performance levels which are comparable to those of the linear and the rank detectors. Surprisingly, even in environments which are heavy tailed such as the shrimp and seismic noise, the sign detector had a relatively poor performance. It was expected that the sign detector would have a better performance level than either the rank or linear detectors, though this was not the case. It is important to emphasize that for complex and non-stationary environments, such as those examined, either the rank detector or the quantizer detector may be the wisest choice from the standpoint of robustness and simplicity. The rank detector is particularly attractive since no optimization need be done to determine the number of levels or the spacing between levels.

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